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The Paradoxical Relationship between Econometric Effectiveness and Legal Certainty

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The Paradoxical Relationship between Econometric Effectiveness and Legal Certainty

Abstract: Be it information asymmetry in banking regulation or game theory as applied to child custody in divorce, economic theory plays an increasingly prominent role in legal scholarship. Econometrics brings welcome empiricism to our efforts to understand the operation both of markets and of society in general. Its application to the law, however, carries with it the risks of failing to account for certain fundamental contradictions between econometric and legal method.

This paper considers the problem of using econometric data to inform legal decision making, and in particular what may be an irreconcilable clash between effective purposive law and legal certainty. This creates problems for the judge asked to make choices based upon desired purposive outcomes, such as achieving the best returns of creditors or having the highest probability of recovery, in order to achieve a legally certain outcome, which is to say that the informed observer is reasonably able to predict the result. Although this paradox is described in the context of insolvency law, it is inherent to legal method in general rather than being specific to this discipline.

The exact nature or desirability of either legal certainty or effective social engineering through the law is beyond the scope of this paper, as is the question of the extent to which legal reasoning is genuinely deductive. The purpose is to demonstrate that the objectives of certainty and effectiveness are pulling in opposite directions: an effective law is uncertain, and a certain law is ineffective. The lawmaker must choose between one and the other.

Keywords: Legal Certainty, Legal Reasoning, Purposive Law, Econometrics, Insolvency Law

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The Paradoxical Relationship between Econometric Effectiveness and Legal Certainty

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I. INTRODUCTION

Be it information asymmetry in banking regulation or game theory as applied to child custody in divorce, economic theory plays an increasingly prominent role in legal scholarship. Econometrics brings welcome empiricism to our efforts to understand the operation both of markets and of society in general. Its application to the law, however, carries with it the risks of failing to account for certain fundamental contradictions between econometric and legal method.

This paper considers the problem of using econometric data to inform legal decision making, and in particular what may be an irreconcilable clash between effective purposive law and legal certainty. This creates problems for the judge asked to make choices based upon desired purposive outcomes, such as achieving the best returns of creditors or having the highest probability of recovery, in order to achieve a legally certain outcome, which is to say that the informed observer is reasonably able to predict the result. Although this paradox is described in the context of insolvency law, it is inherent to legal method in general rather than being specific to this discipline.

I will do this by creating a model of a deductive judge, asked to make choices based upon desired purposive outcomes in order to achieve a legally certain outcome. The term ‘model’ in this case does not refer to a ‘model law’ in the traditional jurisprudential sense but rather an economic model: an abstraction of the problem where, accepting certain assumptions to be true, and holding all other things to be equal, we will analyse the relationship between effectiveness and certainty.

Where I use the term “effective” I mean a test based on a quantifiably significant factor, being something that has a measurable impact on the phenomena. Where I use the term “certain” I mean a test that provides consistent, predictable results. The exact nature or desirability of either legal certainty or effective social engineering through the law is beyond the scope of this paper, as is the question of the extent to which legal reasoning is genuinely deductive. The purpose is to demonstrate that the objectives of certainty and effectiveness are pulling in

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opposite directions: an effective law is uncertain, and a certain law is ineffective. The lawmaker must choose between one and the other.

II. Legal Certainty

Whether a cornerstone of Dicey’s rule of law protecting individuals from an arbitrary state\(^1\), or a principle of inherent morality in Fuller’s moral law\(^2\), whether debated, rejected, or endorsed, the concept of legal certainty has always been a key component of any discussion of the principles of proper law making.\(^3\) As full engagement with the worthy topic of the nature of legal certainty is beyond the scope of this paper, it is to be hoped we can uncontroversially summarise by suggesting that by certainty we mean that the law should be predictable. Before we arrive at the court we should have a good idea of what the outcome will be: the murderer will be convicted of murder, the thief convicted of theft.

“Lawyers and clients are able to predict what the outcome of a particular legal question is likely to be in the light of previous judicial decisions. Also, once the legal rule has been established in one case, individuals can orientate their behaviour with regard to that rule, relatively secure in the knowledge that it will not be changed by some later court.”\(^4\)

We can identify two convictions central to this notion of certainty; first, that the rule of law is preserved, protecting us from the whims of the judiciary; second, that we can safely order our actions according to the probable legal response:

“A man may have as bad a heart as he chooses, if his conduct is within the rules.

In other words, the standards of the law are external standards, and, however much it may take moral consideration into account, it does so only for the purpose of drawing a line between such bodily motions and rests as it permits, and such as it does not. What the law really forbids, and the only thing it forbids, is the act on the wrong side of the line, be that act blameworthy or otherwise.”\(^5\)

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Encoded European Law principles like clarity, calculability and reliability\(^6\) are all the progeny of the desire for legal certainty, as is the doctrine of *stare decisis* in the common law\(^7\). In contemporary insolvency law we see a repeated focus on the creation of a “clear, predictable and transparent insolvency process which enables both debtor and creditor to calculate the consequences in the event insolvency actually occurs.”\(^8\) Promoting legal certainty in this form has been a pillar of the advice given to developing nations by organisations like the IMF, the World Bank and the OECD\(^9\), the reasoning being that in a legally certain insolvency regime, potential creditors will be able to better evaluate the risks, encouraging economic investment and stable development.

### III. How do we achieve Legal Certainty?

Lawyers have two important techniques with which to achieve consistency between different cases.

Deductive reasoning operates from the general to the particular, on the premise that if individual premises are true then the conclusion cannot be false, and is central to the civil law. It finds “its simplest and yet most powerful expression in the Aristotelian syllogism”\(^10\), a technique that is central to the French *syllogisme judicaire* and via France is at the heart of civil law\(^11\).

Reasoning by analogy, or reasoning from part to part\(^12\), looks for similar facts in other cases and is essential to the doctrine of precedent in the common law system: “It is extremely unlikely that judges will find an authority which corresponds precisely to the facts of the case before them. What they have to do is find an analogous case and use its reasoning to decide the case before them.”\(^13\) As Oliver Wendell Holmes observed, “It is something to show that the consistency of a system requires a particular result, but it is not all. The life of the law has not been logic, it has been experience.”\(^14\)

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\(^7\) Slapper & Kelly, p75


\(^10\) Slapper & Kelly, p187


\(^12\) Slapper & Kelly, p189

\(^13\) Slapper & Kelly, p190

Regardless of their differences, or even the extent to which both methods are used in either or both legal systems, what they have in common is the process of separating data into parts, or “facts”, and testing them, whether for their logical consistency or their similarity. This quality of separation is an important contrast to econometric method, as we shall see below. Before we can do this there is another form of reasoning that should be considered. Holmes’ passage quoted above continues:

“The felt necessities of the time, the prevalent moral and political theories, intuitions of public policy, avowed or unconscious, even the prejudices which judges share with their fellow men, have had a good deal more to do than the syllogism in determining the rules by which men should be governed.”15

Inductive reasoning goes from the part to the whole, the particular to the general, and is in this sense the opposite of deductive reasoning. Some critiques of legal method as a form of deductive reasoning focus on the role of its opposite:

“The use of inductive reasoning cannot claim the certainty inherent in the use of deductive reasoning. The introduction of this increased element of uncertainty is inescapable and unconscious, but it is also appropriate to note that the determination of precedent by later courts gives the later judges scope to consciously manipulate precedents... Thus, the apparent deductive certainty of the use of precedent is revealed to be based on the much less certain use of inductive reasoning and reasoning by analogy, with even the possibility of personal views of the judges playing some part in deciding cases. This latter factor introduces the possibility that judges do not in fact use any form of logical reasoning to decide their cases, but simply deliver decisions on the basis of an intuitive responses to the facts of the case and situation of the parties involved.”16

For the purposes of a model to describe the tension between certainty and effectiveness we shall concentrate on the judge as a creature of deductive reasoning striving to produce objectively predictable results. Even if we were to argue that judicial reasoning were entirely inductive, this still wouldn’t exclude impact of the desire to act or give the semblance of acting deductively. Furthermore, as described above, the inductive judge is a legally uncertain judge in the first place. This does open the door to the possibility that an inductive judge is free to enact ‘effective’ law in a way a deductive judge is not, but this does no harm to our experiment.

15 ibid
16 Slapper & Kelly, p191
IV. ECONOMETRICS AND LAW

Econometrics is the application of quantitative method to elucidate and test economic theory. Quantification is, essentially, gathering things together and then measuring them. At its heart is the notion that trends identified within groups can be applied to predict trends in similar groups: if 80 of a group of 100 English men in their thirties say they will vote Conservative, then we can predict that 80% of Englishmen in their thirties will vote Conservative.

Let us imagine that we have discovered a strong trend showing that Conservative voters were very bad at running businesses. Let us also imagine that our judge is privy to this information. If he is then required to decide if a business is likely to recover, and all other things being equal, he might reasonably decide when confronted by an Englishman in his thirties that it was best not to grant him another chance at saving his failing business.

There does not appear to be a problem in unifying these two types of reasoning to produce a superior commercial law. By liquidating businesses run by Englishmen in their thirties, the econometric judge has made the law more effective through the application of relevant empirical evidence. He also appears to have made the law more certain. That which can be quantified can also be verified: in order to have been able to count it in the first place you had to be able to test it. In our example it would not be especially difficult to deduce and test the age and nationality of the owner of the business. On this basis we can hypothesise a positive relationship between effectiveness and certainty: the more empirically effective the law is, the more certain we can be of the outcome before we go to trial:

![Graph showing the relationship between certainty and effectiveness](image)

You will recall from the introduction that ‘effectiveness’ here refers to a test based upon quantifiably significant factor and ‘certainty’ to a test that produces a predictable outcome. The rest of this paper is intended to demonstrate why this apparently reasonable hypothesis of a positive relationship is wrong.
V. The Paradox of Power and Replication

The problem for the judge is that he is making his observations in what is known as a ‘low power scenario.’ Power is the probability that where a relationship or effect exists, you will observe it to exist; it is the probability that your statistical test will yield statistically significant results. You do not need to understand the nuances of power to follow the argument in this section, beyond that generally something will have a low power where you have a low number of samples.

A judge is not in a position to evaluate probable outcome as he can only make his observations in the time honoured fashion: one case at a time. Determining probability requires grouping things together so that the variables interact. No matter how much gamblers believe in winning streaks, the chance of a second coin toss being a head is in no way affected by the first. We will recall that this difference between separation and grouping was a fundamental difference between deductive legal reasoning and quantitative analysis.

Intuitively, this still feels like it shouldn’t be a problem. If our econometrists have provided enough data about factors likely to cause failure, then we should be able to use them as guidelines to help us make consistent decisions. Unfortunately this is where we run into the paradox of power, described in 1996 by the statistician Ottenbacher when observed that the “apparently paradoxical conclusion is that the more often we are well guided by theory and prior observation, but conduct a low power study, the more we decrease the probability of replication!” 17

The probability of replication he talks about here is the chance that given the same data, the same facts, we reach the same conclusion. This is another way of asking: how certain is the test? How likely are we to know the result before we conduct the test? We can show how Ottenbacher’s paradox might impact upon legal certainty by applying a power test to a hypothetical model insolvency law.

VI. The Fictional Insolvency Act 2010

Under the Fictional Insolvency Act 2010, any judge who receives a request from a creditor of an insolvent firm to liquidate the company is given the option to either a) immediately liquidate, or b) offer a 12 month moratorium during which the company has a chance to fix its problems, during which time he will not be required to pay his debts.

The judge must decide based upon which option is most likely to provide the best returns to creditors. By the best returns we mean the largest proportion of the outstanding debt. Although this system is clearly a huge simplification, the ‘best returns to creditors’ standard is common to most developed insolvency law systems, and credit moratoriums are a common option within packages of rescue solutions. Remember he can only pick the option most likely to give the best returns, as he can’t know which one will give the best returns until after he makes his decision. This will be considered to be a “good” law if it is both effective in providing the best returns to creditors, and certain so that commerce can model its behaviour accordingly.

In our model the judge has an either/or decision to make based upon his evaluation of which is most likely to achieve a particular result. This is as opposed to an either/or decision about something which has happened in the past or which currently exists, for example; did the defendant strike the claimant, or does the claimant have title to the property? The two types of questions have a substantive conceptual difference; the difference between making a decision about the certain (something that has already happened, however much we may reconstruct the event in the court), and making a decision about the uncertain (something that may happen in the future, and which cannot be made to exist until it actually happens). This is the difference between asking “did Keith win the lottery” and “will Keith win the lottery if he buys a ticket.” This is important to remember when we come to the maths: we are dealing with probable outcomes.

Our judge must look at the facts of the case before him, and determine if there is some element which he recognises as being decisive. It is important to note that this is just a model of a judge: how a fictional judge might act if the only factors he can consider are repetition and certainty, and his only method is deduction. This is, of course, not an accurate representation of how a real life judge makes his decisions, but a technique for illustrating the methodological clash. We can reproduce the model judge’s dilemma as a diagram.
We have given our model judge two relationships that he might look for to make his decision.

The first relationship (X) exists in 40% of the cases that the judge evaluates, and is an effective relationship: it does determine whether the business will succeed or fail.

The second (Y) exists in 5% of the businesses that the judge evaluates, and actually has no effect on whether the business is better rescued or liquidated. 5% is the standard minimum acceptable error in social science research, which is to say that if a social scientist observed a correlation of less than 5% conventionally he would reject it as spurious.

The judge does not know whether these two factors are effective or not. So he will experiment with both, and choose the one which provides the best replication: the one which appears to give the most consistent results.

So, which of these tests provides the most legally certain result: the effective or the spurious? We know that relationship X is causal, and that relationship Y is not. The judge is not so fortunate as to have this information, so how does he determine which relationship to use? In social science one groups results together to evaluate them across the whole. A judge must evaluate one case at a time. This is the difference between a scientist who does one experiment with two samples, and another who does two experiments with one sample at a time, or two hundred samples against two hundred separate experiments. The difference between these two may not be intuitively obvious, but we can demonstrate it mathematically.
VII. Probability of Replication in Spurious and Non-Spurious Scenario

Let us compare what happens in two scenarios; the first where the judge looks for the existence or non-existence of X, the effective relationship with a 40% correlation, in two cases in a row.

A. SCENARIO 1: PROBABILITY OF REPLICATION WHERE THERE IS AN EFFECTIVE RELATIONSHIP X

| P1(reject|E,1) * P1(reject|E,2) = 0.4²  | = 0.16 |
| P2(¬reject|E,1) * P2(¬reject|E,2) = (1-0.4)²  | = 0.36 |

What does this mean? We are looking at the probability that looking for relationship X in two cases in a row will get the same result both times (and thus be “certain”).

This paragraph is for the reader unfamiliar with statistical notation. “P” is probability, and the number next to it refers to the relevant box on the diagram above. “Reject” is the rather awkward double negative used in statistics to mean “rejects the null hypothesis”, which is to say that we reject that the hypothesis is false, or to put it another way, that the relationship exists. “¬” is the mathematical symbol for “not”, so “¬reject” means “does not reject the null hypothesis”, which is a triple negative meaning that the relationship has not been observed. “E” is effect, which means that the relationship is an effective relationship (it is a relationship where the factors are influencing each other, as opposed to one where two things just happen to be going on at the same time), and the number after the comma is the number of the observation. So “P(¬reject|E,2)” means the probability that the observer does not observe an existing relationship in the second case.

What we see here is that with a 40% chance of observing the phenomena, we have a 52% chance of getting the same result twice (either two negatives or two positives) if we perform two observations separately. This is quite different from the 64% chance we have if we observed the two cases together like a social scientist. Where does this 64% come from? When you group cases together the probability of observing the phenomena accumulates. So there is the 40% chance of observation from the first case, and in the other 60% remainder is another 40% chance of observation (40% of 60 is 24, and our first 40 plus our new 24 is 64). The difference between two observations at the same time and two observations in a row is this “12%” – a difference that gets bigger the larger the sample is - because cases observed together accumulate as evidence, whereas cases analysed separately can only be compared for consistency. The judge’s inability to accumulate simultaneous data forces him to rely on separate comparisons of consistency: a court of law is an extremely poor laboratory.
B. Scenario 2: Probability of replication based on spurious relationship Y

\[
\begin{align*}
P_3(\text{reject}|\neg E,1) \times P_3(\text{reject}|\neg E,2) &= 0.05^2 = 0.0025 \\
P_4(\neg\text{reject}|\neg E,1) \times P_4(\neg\text{reject}|\neg E,2) &= (1-0.05)^2 = 0.9025
\end{align*}
\]

Where we base our decision making on spurious data that is only observed in 5% of cases there is a 91% chance of replication. Or, to put it another way, the judge finds that making his decision using observations of a relationship that has no effect (Y) is almost twice as likely to produce the same result as using his observations of an effective relationship (X). The spurious data is more certain than the effective data.

VIII. Impact on Replication of Higher Probable Correlation

The first question you might be asking about this model is this: why only 40%? This may seem a much less replicable event than, say, the 80% for English insolvents I described in my earlier example. If we do the mathematics again we find that it doesn’t have as big an impact as we might expect:

A. Probability of replication of observation of effective relationship at 80% incidence

\[
\begin{align*}
P_1(\text{reject}|E,1) \times P_1(\text{reject}|E,2) &= 0.8^2 = 0.64 \\
P_2(\neg\text{reject}|E,1) \times P_2(\neg\text{reject}|E,2) &= (1-0.8)^2 = 0.04 \\
&= 0.64
\end{align*}
\]

This is only 12 points better than the 40% correlation, which is a pretty dismal return for doubling the chance of observation! It also gives a strong hint as to the actual shape of the correlation between effectiveness and certainty, and leads us to our next figure that is worth analysing: a relationship with a 95% chance of being observed. Here, of course, the probability of repetition is exactly equal. What we have is a practical inversion: the judge acting principally on an observation of the existence of an effective relationship (and occasionally on when it is not there) is exactly as certain as the judge acting on the failure to observe a spurious relationship (and occasionally when it is there). Which means that the judge, left only with replication as a means to evaluate, still cannot distinguish between the effective and the spurious. This is not the only problem with the 95% correlation, but before considering that we should look at the impact of continued replication.
IX. Divergence of Probabilities through Repeated Replications

The problem becomes serious as we increase the number of replications (in this model the number of cases the judge sits on)

A. Probability of replication over three cases based upon the effective relationship X:

\[
\begin{align*}
&P1(\text{reject}|E,1) \times P1(\text{reject}|E,2) \times P1(\text{reject}|E,3) = 0.4^3 = 0.064 \\
&P2(\neg\text{reject}|E,1) \times P2(\neg\text{reject}|E,2) \times P2(\neg\text{reject}|E,3) = (1-0.4)^3 = 0.216 \\
&\quad = 0.28
\end{align*}
\]

B. Probability of replication over three cases based upon the spurious relationship Y:

\[
\begin{align*}
&P3(\text{reject}|\neg E,1) \times P3(\text{reject}|\neg E,2) \times P3(\text{reject}|\neg E,3) = 0.05^3 = 0.000125 \\
&P4(\neg\text{reject}|\neg E,1) \times P4(\neg\text{reject}|\neg E,2) \times P4(\neg\text{reject}|\neg E,3) = (1-0.05)^3 = 0.857375 \\
&\quad = 0.8575
\end{align*}
\]

Although both probabilities have reduced, the spurious relationship is now four times more likely to be replicated than the effective one. Where the only means of evaluation is replication the spurious data rapidly becomes more credible: the more cases the judge sees the more the two relationships probability diverges and the more relatively certain the spurious data appears.

What does this mean? If we have two judges, one who uses the actual, effective relationship to determine whether to rescue or liquidate the company, and another who uses the spurious relationships, then it is the effective judge who will produce uncertain results. Because we only have replicability to judge the quality of his decisions on, his decisions will appear to be bad. The other judge, whose decision is based upon a relationship which has nothing to do with what actually impacts on business failure, produces more predictable results. If a good law is a certain law, the more spurious the relationship upon which the law is based the better the law is.

X. Goldfish and Billionaires

It is possible to illustrate this paradox using slightly less mathematics.
A judge sits on insolvency cases.

40% of the time, the business is run by a billionaire and the creditors do better if the business is rescued.
5% of the time, the business has goldfish in the lobby and the creditors do better if the business is rescued.

The first one is a real impact; the billionaire owner really helps the creditors. The second one is spurious; goldfish do not save businesses, although very occasionally rescued businesses have goldfish.

The judge, however, has no way of knowing which one is or is not spurious, and can only rule one case at a time, basing his decision on which relationship appears to be most consistent.

In what is known as a “low power” situation, which a court case is the most extreme possible example of, spurious relationships will repeat more often. This means the judge gets a more consistent result if he bases his decisions on the existence of goldfish than on the presence of a billionaire.

This analogy is a rather brutal simplification of the paradox and it is best not to stretch it too far, so do not spend too much time thinking about which one is the billionaire and which the goldfish. The root of the problem is to do with contrasting methodologies, not specific probabilities and types of causal relationship. However, the analogy hopefully makes accessing the issues more straightforward for readers uncomfortable with the maths.

**XI. Is Econometrics still useful for law?**

If extrapolate our numbers to illustrate the relationship between effectiveness (testing using quantifiably significant factors) and certainty (testing to provide predictable results) that we have discovered, it looks like this:
This curve shows the relationship between certainty and effectiveness after ten iterations – after the judge sits on ten cases. The more repetitions occur, the more this U shape broadens, exacerbating the rapid fall in certainty as we move away from the extremes of effectiveness. Instead of the positive relationship between effectiveness and certainty that we expected, what we find is that effective relationships only make certain tests when they occur almost all the time.

To make matters worse, we should consider that if some factor existed that 95% of the time could be observed to determine whether a business should be rescued or not, the market would have adapted to eliminate it; banks do not lend money to the criminally insane, for example, or to people who are dead. This means that except for the most bizarre cases we have to practically exclude the extremely effective relationships, leaving only the spurious ones as highly certain!

Businesses fail for a wide array of rapidly changing reasons, so even if we were able to accurately map effective correlations for business failure they would not be absolute. Effective business operates somewhere in the middle of the chart. The reason effective relationships appear inconsistent is because effective relationships are inconsistent.

Does this mean that we cannot use econometrics to inform legal decision making? The science of economics has been of immeasurable value to the development of our financial systems: the problem is not in the economics but how we are trying to use the economics. As Ottenbacher warns us, “the literature with low statistical power is not only committing a passive error, but can actually contribute to diverting attention and resources in unproductive directions.”¹⁸

If we determine an effective relationship between goldfish and business failure then writing a law which prohibits the rescue of businesses with goldfish is a practical use of the data. Asking the judge to determine whether the presence of goldfish is likely to prevent successful business rescue is not. A law which asks the judge to evaluate if a proposal will probably meet an external target will not be legally certain, and a law which is legally certain will exclude the possibility of the judge applying his experience to determine if the ruling is likely to work or not. The clash between quantitative and deductive method means that we can have law that asks the judge to apply external knowledge to predict the probable outcome, or law that is legally certain, but not both at the same time.

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